Definite Integral

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \overbrace{f(x_i^*)\Delta x_i}^{a_i}$$

Properties of Sums

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n$$

each a_i is a real number

① $$\boxed{\sum_{i=1}^{n} c\cdot a_i = c\sum_{i=1}^{n} a_i}$$

$$\left(= ca_1 + ca_2 + ca_3 + \cdots + ca_n = c(a_1 + a_2 + \cdots + a_n)\right)$$

$$4x^3 + 4x^2 + 4x + 4$$

② $$\boxed{\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i}$$

$$(x+y) + (x^2+y^2) + (x^3+y^3) = (x+x^2+x^3) + (y+y^2+y^3)$$

③ $$\boxed{\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i}$$

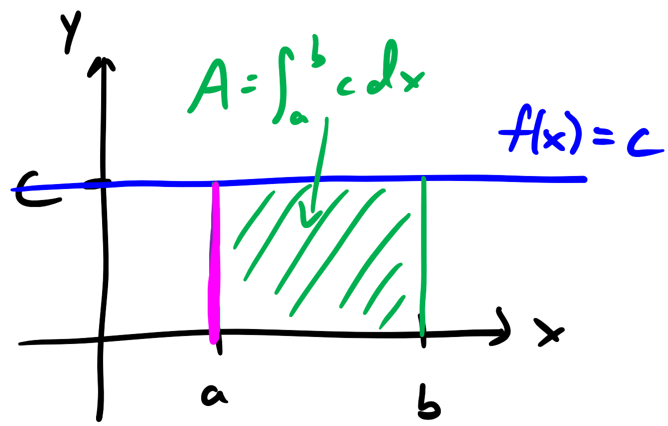
Properties of the Definite Integral

f, g - functions, c is a constant

① $$\int_a^b c\, f(x)\, dx = c \int_a^b f(x)\, dx$$

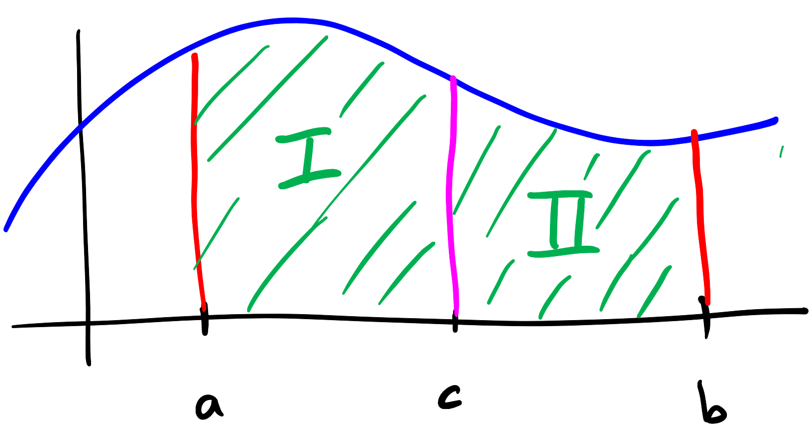
② $$\int_a^b \big(f(x) \pm g(x)\big)\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$$

③ $$\int_a^b c\, dx = c(b-a)$$



④ $$\int_a^a f(x)\, dx = 0$$

⑤ $$\underbrace{\int_a^c f(x)\, dx}_{\mathrm{I}} + \underbrace{\int_c^b f(x)\, dx}_{\mathrm{II}} = \underbrace{\int_a^b f(x)\, dx}_{\mathrm{I}+\mathrm{II}}$$



$$\int_b^a \qquad \Delta x = \frac{a-b}{n}$$

⑥ $$\int_b^a f(x)\, dx = -\int_a^b f(x)\, dx$$

⑦ If $f(x) \geq 0$ for $a \leq x \leq b$

then $\int_a^b f(x)\,dx \geq 0$

⑧ If $f(x) \geq g(x)$ for $a \leq x \leq b$

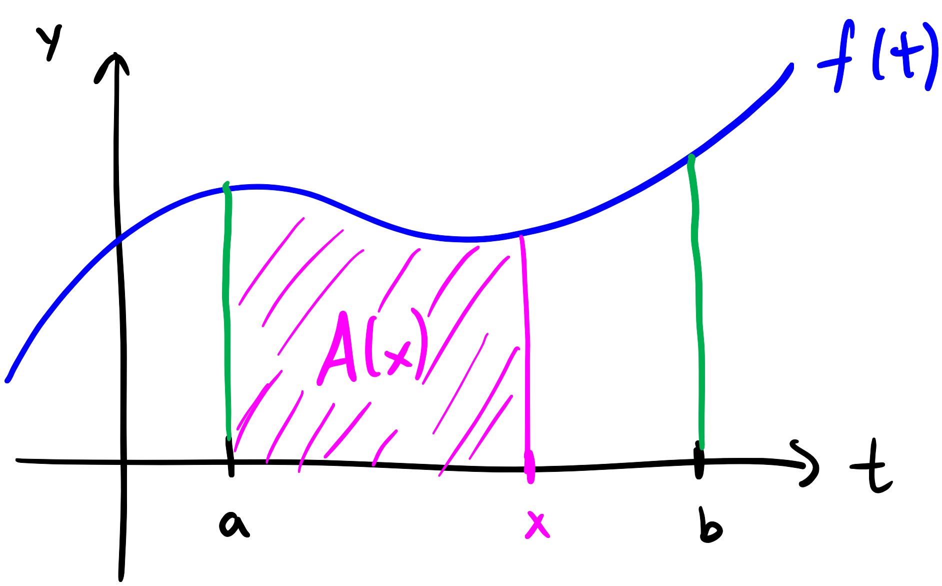
then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

⑨ If $m \leq f(x) \leq M$ for $a \leq x \leq b$

then $$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

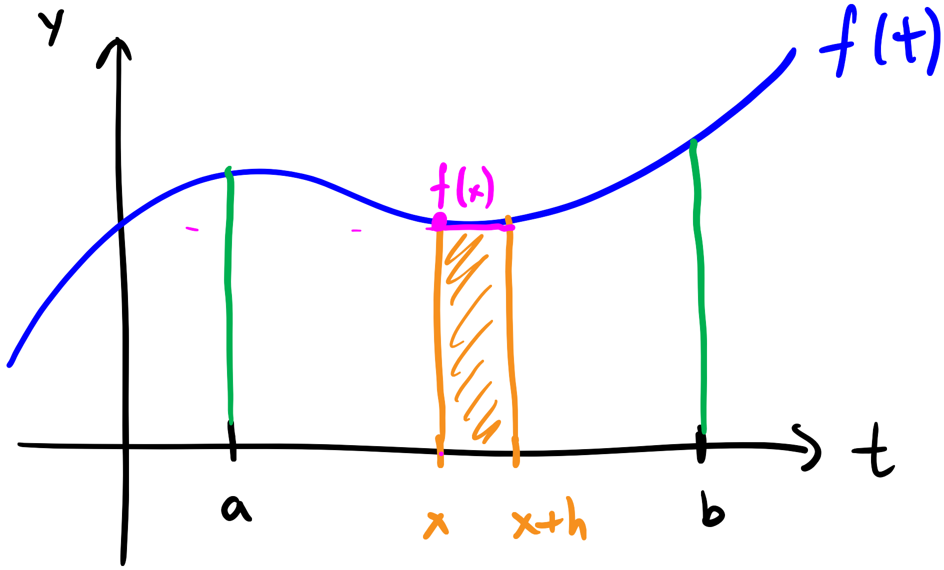
Let $f(x)$ be a continuous function on $[a,b]$. Define a new function

$$A(x) = \int_a^x f(t)\,dt$$



Total area

$A(b) = \int_a^b f(t)\,dt$



$$A(x+h) - A(x) \approx h f(x)$$

(divide by h)

$$\hookrightarrow \frac{A(x+h)-A(x)}{h} \approx f(x)$$

(take lim as $h \to 0$)

$$\hookrightarrow A'(x) = \lim_{h \to 0} \frac{A(x+h)-A(x)}{h} = f(x)$$

$$A'(x) = f(x)$$

The Fundamental Theorem of Calculus (pt. I)

If f is continuous on $[a,b]$, then the function $g(x) = \int_a^x f(t)\,dt$, $a \leq x \leq b$

is continuous on $[a,b]$ and differentiable on (a,b), moreover $g'(x) = f(x)$.

In particular, $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$. That is, $\int_a^x f(t)\,dt$ is an antiderivative of $f(x)$.